A Flying Qualities Criterion for the Design of Fighter Flight-Control Systems

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It is readily apparent that current longitudinal flying qualities criteria do not adequately account for the effects of dynamic modes introduced by today's complex flight control systems (FCS). To remedy this situation, a combined analytical and experimental investigation was recently conducted using the U. S. Air Force/Cornell Aeronautical Laboratory (USAF/CAL) variable-stability T-33 airplane. Based on an extensive pilot-in-the-loop analysis of the experimental results, a design criterion was developed which is shown to be applicable to a wide range of short-period and FCS dynamics. A simplified version is also presented to provide the designer with preliminary estimates of flying qualities.

Nomenclature

BW	=	bandwidth, the frequency at which the phase
		angle of the θ/θ_c transfer function = -90°,
		rad/sec
$(BW)_{\min}$	==	value of the closed-loop bandwidth which the
, , , , , , , , , , , , , , , , , , , ,		pilot is trying to achieve in precision tracking
		tasks, rad/sec
db	_	decibel units for Bode amplitude, where amplitude
		in decibels = $20 \log_{10}$ (amplitude)
$(dA/d \not \downarrow)_{ad}$	_	rate of change of Bode amplitude with phase for
() /- /- /		the airplane plus pilot time delay at $\omega =$
		$(BW)_{\min}$, db/deg
F_S	==	elevator stick force, positive for a pull, lb
F_S^S/n		steady-state stick force per unit normal accelera-
1 8/10		tion change, at constant speed, lb/g
K_p	==	steady-state pilot gain, lb/deg
K_{θ}		gain of airplane's θ/F_S transfer function, (deg/
11.0		sec)/lb
n		normal acceleration at center of gravity, positive
,,		for a pullup, g
n/α	_	steady-state normal acceleration change per unit
π, α		angle-of-attack change, when the airplane is
		maneuvered at constant speed, g/rad , $n/\alpha \approx$
8	_	$(\overline{V}_T/g)(au_{ heta_2})^{-1}$
8 V:		$(V_T/g)(au_{ heta_2})^{-1}$ Laplace operator
$V_{ m ind}$	===	$(V_T/g)(au_g)^{-1}$ Laplace operator trimmed indicated airspeed, knots
$V_{ ext{ind}} \ \zeta_{\scriptscriptstyle SP}$	=	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio
$V_{ ext{ind}} \ ec{\zeta}_{SP} \ ec{\zeta}_3$	=	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag
$V_{ ext{ind}} \ \zeta_{\scriptscriptstyle SP}$	=	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon,
$V_{ ext{ind}}$ ζ_{SP} ζ_{3} $ heta$	=======================================	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad
$V_{ ext{ind}} \ ec{\zeta}_{SP} \ ec{\zeta}_3$	=======================================	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude,
$V_{ ext{ind}}$ ζ_{SP} ζ_3 θ	=======================================	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad
$V_{ ext{ind}}$ ζ_{SP} ζ_{3} $ heta$	=======================================	$(V_T/g)(\tau_{\theta_c})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch
$V_{ ext{ind}}$ ζ_{SP} ζ_3 θ	=======================================	$(V_T/g)(\tau_{\theta_c})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c	=======================================	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad
$V_{ ext{ind}}$ ζ_{SP} ζ_3 θ	=======================================	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus con-
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e		$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c		$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus conopen-loop transfer function of airplane plus con-
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e θ/F_S θ/θ_e	=======================================	$(V_T/g)(\tau_{\theta_c})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus control system plus pilot
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e	=======================================	$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus control system plus pilot closed-loop transfer function of airplane plus
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e θ/F_S θ/θ_e		$(V_T/g)(\tau_{\theta_c})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus control system plus pilot closed-loop transfer function of airplane plus control system plus pilot
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e θ/F_S θ/θ_e		$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus control system plus pilot closed-loop transfer function of airplane plus control system plus pilot Bode amplitude of any resonant peak in the θ/θ_c
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e θ/F_S θ/θ_c θ/θ_c θ/θ_c		$(V_T/g)(\tau_{\theta_c})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus control system plus pilot closed-loop transfer function of airplane plus control system plus pilot Bode amplitude of any resonant peak in the θ/θ_c transfer function, dB
$V_{ m ind}$ ζ_{SP} ζ_3 θ θ_c θ_e θ/F_S θ/θ_e		$(V_T/g)(\tau_{\theta_2})^{-1}$ Laplace operator trimmed indicated airspeed, knots short-period damping ratio damping ratio of second-order control-system lag airplane's pitch attitude with respect to horizon, positive nose up, deg or rad commanded change in airplane pitch attitude, deg or rad $(\theta_c - \theta)$, error between the commanded pitch attitude and the airplane pitch attitude, deg or rad transfer function of θ to F_S for airplane plus control system open-loop transfer function of airplane plus control system plus pilot closed-loop transfer function of airplane plus control system plus pilot Bode amplitude of any resonant peak in the θ/θ_c

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= time constant of control system lag element, sec = time constant of pilot's lead element, sec

 τ_2

 $au_{\theta 2} = ext{airframe lead time constant in } heta/F_S ext{ transfer function, sec}$ $\omega_{SP} = ext{short-period undamped natural frequency, rad/sec}$

= time constant of pilot's lag element, sec

ω₃ = undamped natural frequency of second-order control-system lag, rad/sec
 Δ = phase angle of the simplementure plus pilot time delay at α =

 $\not \downarrow_{ad}$ = phase angle of the airplane plus pilot time delay at $\omega = (BW)_{\min}$, deg

 \swarrow_{pc} = phase angle of the pilot compensation at $\omega = (BW)_{\min}$, deg

I. Introduction

IN recent years, complex flight control systems (FCS), employing various combinations of feedback and feedforward loops, have become increasingly common. Many such systems being designed and tested today introduce additional dynamic modes that have characteristic frequencies of approximately the same magnitude as the short-period frequency. In these cases, characterizing the response by the short-period dynamics alone is often difficult and loses meaning. The effects of the higher-order control-system dynamics can produce airplane longitudinal flying qualities that are completely unacceptable, even if the short-period mode itself is well behaved.

Because of the significance of control-system dynamics, it is important that new criteria for the design of FCS be developed, to insure satisfactory maneuvering characteristics. The preliminary development of such criteria is the subject of this article.

II. Effects of Control-System Dynamics on Flying Qualities

To develop meaningful design criteria that consider the effects of FCS dynamics, a logical first step is to conduct experiments to determine, in detail, the flying qualities problems that can be caused by such dynamics. In 1967, the effects of higher-order system (HOS) dynamics on fighter longitudinal flying qualities were investigated in a preliminary ground simulator experiment, followed by an in-flight experiment. Based on a study of the results of these experiments and a survey of current FCS designs for fighter aircraft, an inflight experiment was recently completed by the authors 4 to provide a broad base of data on the effects of FCS dynamics for development of criteria. All of these experiments employed the USAF/CAL variable stability T-33 airplane shown in Fig. 1.

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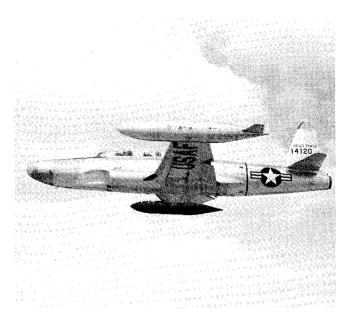


Fig. 1 USAF/CAL variable-stability T-33 airplane.

The HOS flight program² simulated the effects of control system modes having natural frequencies higher than the short-period natural frequency. The order of these modes varied from fourth to seventh order. The airplane's maneuvering response for these configurations looked very much like the short-period response plus a pure time delay. Most of the pilot's difficulties in flying such configurations were related to the magnitude of the time delay and the rapidity of the response following the delay. For the extreme cases, violent pilot-induced oscillations (PIO's) occurred whenever precision attitude tracking was attempted.

The primary purpose of the present experiment was to evaluate the effects of a single FCS zero $(1/\tau_1)$ and a single FCS pole $(1/\tau_2)$ on eight basic short-period configurations. A second-order FCS complex root was also included, but its natural frequency (ω_3) was fixed at 63 rad/sec for most of the experiment. The block diagram of Fig. 2 represents how the pilot would view the pitch attitude response to stick force inputs for the configurations simulated. The actual configurations simulated are shown in Fig. 3. Unlike the HOS program, the shape of the airplane's maneuvering response was completely altered by the FCS dynamics shown here.

In the present flight program, the evaluation pilot was asked to fly a series of VFR and IFR tasks, with the emphasis on the VFR tasks, which were designed to be representative of the air-superiority fighter mission. The tasks included the acquisition and tracking of visual targets, symmetrical pullups and pushovers, rapid high-g turn reversals, and attitude command tracking tasks (IFR). In addition, samples

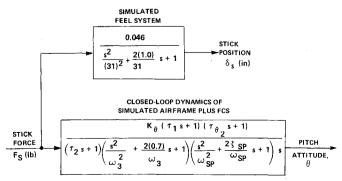


Fig. 2 Block diagram for the configurations simulated in present experiment.

of the preceding tasks were performed in the presence of random noise inputs. After performing these tasks, each evaluation pilot was asked to make recorded comments on specific characteristics of the configuration. As part of these comments, the pilot assigned an over-all numerical rating using the Cooper-Harper Pilot Rating Scale.⁵ The evaluation pilots were allowed to select the stick-force-to-elevator gearing, and therefore the F_s/n , for each evaluation configuration.

For the configurations with good damping ($\zeta_{SP} \simeq 0.7$) at each flight condition, it was found that the proper amount of lag in the FCS could be beneficial when the short-period frequency was high. However, when the short-period frequency was medium or low, lag was always degrading. Conversely, the proper amount of lead could be beneficial when the short-period frequency was low, but was degrading otherwise. For the two medium-frequency, poorly damped, short-period configurations evaluated at the low-speed flight condition, the effect of FCS lag was to smooth out the short-period oscillations somewhat. However, flying qualities with lag were much worse than without lag. As a matter of fact, the lag caused some rather serious pilot-induced oscillations

Fairly early in the analysis of the results of the present experiment, it became obvious that the effects of controlsystem dynamics on fighter flying qualities were complex and that direct correlations between pilot opinion and particular parameters in the FCS and airplane transfer functions would be difficult to obtain. The pilot comment data indicated that the pilot evaluates the total response of the airplane to his inputs and is not concerned with, or often aware of, the characteristics of the individual elements that combine to produce that response. A short-period flying qualities criterion for fighter aircraft, called the C* criterion,6 does impose requirements on the total response to pilot inputs. To determine whether the C^* criterion accounts for the effects of significant FCS dynamics, the configurations evaluated in the present experiment were compared with the C^* criterion boundaries.

The eight basic short-period configurations, those with negligible FCS dynamics, correlated with the C^* criterion boundaries fairly well. This is not surprising, however, since the C^* envelope boundaries are based on data for which the FCS dynamics were also negligible. Disagreements with the C^* boundaries were caused by the effects of FCS dynamics, the very effects that the criterion was intended to handle. In fact, at least one control-system configuration in

					SHORT PERIOD CHARAC					TERISTICS		
CONTROL SYSTEM CHARACTERISTICS			n/α = 18.5 g/RAD V_{ind} = 250 KT $1/\tau_{\theta 2}$ = 1.25 SEC ⁻¹					$n/\alpha = 50 \text{ g/RAD}$ $V_{\text{ind}} = 350 \text{ KT}$ $1/\tau_{\theta} = 2.4 \text{ SEC}^{-1}$				
1/ 71	τ ₁ 1/ τ ₂		3	2.2/.69	4.9/.70	9.7/.63	5.0/.28	5.1/.18	3.4/.67	7.3/.73	16.5/.69	
0.5	2	6	3	1A								
0.8	3.3	П			i		ĺ		6A			
2	5	П		1B	2A							
3.3	8						1		68	7A		
5	12	Ш			2C				l			
8	19									7B		
00	00	7	5	1D	2D	3A	4A	5A	6C	7C	8A	
Ш	19	6	3 .	L		<u> </u>				7 D	8B	
Ш	12	Ш			2E	3B	4B	5B				
	8								6D	7E	8C	
	5			_1E	2F	3C	4C	5C	<u> </u>			
	3.3								6E	7F	8D	
	2			1F	2H	3D	4D	5D		7G		
$\sqcup \bot =$	0.8	Ĺ							6F	7H	8E	
<u> </u>	0.5			1G	2J_	3E	4E	5E				
2	5	1	6	1C	2B							
00	5				2G							
♦	2	1	,		21							

NOTE: NUMBERS/LETTERS INDICATE CONFIGURATIONS SIMULATED

Fig. 3 FCS/short-period configurations simulated in present experiment.

each of the eight groups of short-period configurations resulted in a disagreement with the C^* boundaries.

The data were also compared with a number of other open-loop criteria, but with little success. Detailed discussions of these comparisons, including the C^* study, are contained in the final report for the program.³ The difficulty in finding open-loop parameters or criteria with which to correlate all the results of this experiment led to consideration of pilot-in-the-loop analysis.

III. Pilot-in-the-Loop Analysis

Detailed study of the pilot comments from the present experiment showed that the pilots weighted very heavily their ability to acquire a target quickly and track it precisely. Although control of normal acceleration in turns and pullups was also of considerable importance, most of the pilot ratings given during the program appeared to be primarily determined by how precisely the pilot could control the airplane's pitch attitude. This is a typical result for fighter flying qualities evaluations. Because of its importance, control of pitch attitude was examined by use of pilot-in-the-loop analysis.

Mathematical Model

The form of the mathematical model, used in the present analysis to describe pitch attitude tracking, was taken from early work by Systems Technology, Inc. (STI),⁷ and is shown in Fig. 4. The model of the pilot consists of a variable gain (K_p) , a fixed 0.3-sec time delay, and a variable first-order compensation network. The time delay includes the time required for the pilot to sense a change in θ_e , the time required to decide what to do, and the neuromuscular lags. STI⁷ suggests that this time delay should be between 0.2 and 0.4 sec. The value of 0.3 sec chosen for this analysis was an average time delay measured from in-flight tracking records.

It should be noted that the block diagram of Fig. 4 is known as a compensatory tracking model, which means that the pilot operates only on the difference (θ_e) between the airplane's pitch attitude and the commanded pitch attitude. In real life, of course, the pilot also derives information from θ , θ_c , and various motion cues. STI and others have done considerable work since the formulation of this early model, in an attempt to account for these factors. In a crude sense, however, the compensatory model does describe what the pilot is trying to do with the airplane, and it has the advantage that it is relatively simple to analyze. In addition, it appears adequate to explain the more important aspects of attitude tracking, as will be shown in Sec. IV.

Pilot's View of Good Tracking Performance

The first step in the analysis is to identify the performance characteristics that the pilot is trying to achieve when he "adapts" to an airplane configuration. The pilot comment data indicate quite clearly that he wants to acquire the target quickly and predictably, with a minimum of overshoot and oscillation. This suggests that the pilot would like a minimum of phase shift and amplitude attenuation in the closed-loop θ/θ_c Bode plot for frequencies below some fixed value, and no amplitude magnification (resonance) at any frequency.

To express these considerations of closed-loop performance in more precise mathematical terms, the following terms are defined.

Bandwidth (BW): Bandwidth is defined as the frequency for which the closed-loop Bode phase, $\not\lt (\theta/\theta_c)$, is equal to -90° . It is a measure of how quickly the pilot can move the airplane's nose toward the target. (For a simple second-order, closed-loop system, BW would be equal to the system's undamped natural frequency.)

Droop: Droop is defined as the maximum excursion of closed-loop Bode amplitude, $|\theta/\theta_c|$, below the 0db line for

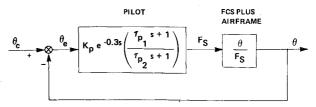


Fig. 4 Mathematical model of pitch attitude tracking.

frequencies less than $\omega = BW$. In the absence of large oscillations, droop is a measure of how slowly the nose settles down on target.

Closed-loop resonance: $|\theta/\theta_c|_{\text{max}}$ is defined as the magnitude of any resonant peak in the closed-loop $|\theta/\theta_c|$ Bode plot. It is directly related to the damping ratio and magnitude of oscillations in pitch attitude which the pilot observes in performing the tracking task.

The next step in the analysis is to put numerical limits on these three parameters, so as to describe a "standard of performance" which the pilot is trying to achieve in performing the required tasks. A helpful guide in determining the performance standards lies in the pilot comments concerning what the pilot does when he cannot achieve good low-frequency performance without causing oscillatory tendencies. When faced with such a tradeoff, the pilot's ratings seem to be primarily a function of the compensation required to achieve a good low-frequency performance (a reasonable bandwidth with a minimum of low-frequency droop) and the oscillatory tendencies that result. In view of these considerations, the following performance standards are assumed for purposes of the present analysis.

1) A minimum bandwidth $(BW)_{\min}$ of 3.5 rad/sec ($\not \in |\theta/\theta_c| \geqslant -90^{\circ}$ at $\omega=3.5$). This value was determined by trying several values of BW in the analysis of the data, until the resulting values of $|\theta/\theta_c|_{\max}$ correlated qualitatively with the pilot comments concerning PIO tendencies.

2) A maximum low-frequency droop of 3 dB $(|\theta/\theta_e| \ge -3)$ dB for $\omega \le BW$. For a simple second-order, closed-loop system with $\zeta = 0.7$, $|\theta/\theta_e| = -3$ dB at $\omega = BW$.

These performance standards are summarized in Fig. 5.

The remainder of the analysis is devoted to determining specific values of K_p , τ_{p_1} , τ_{p_2} which will achieve the performance standards of Fig. 5, with a minimum of high-frequency resonance (low value of $|\theta/\theta_c|_{\rm max}$). The pilot ratings for each configuration should then be a function of K_p , τ_{p_1} , τ_{p_2} , and $|\theta/\theta_c|_{\rm max}$.

Determination of the Required Pilot Compensation

Before the specific values of K_p , τ_{p_1} , τ_{p_2} for a given configuration can be calculated, it is necessary to determine the form of the compensation which the pilot will employ (i.e., whether lead compensation or lag compensation is required). To determine the form of the required compensation, it is logical first to adjust the pilot gain alone, without lead or lag compensation. If a transfer function or a Bode plot is available which describes the θ/F_s dynamics of the complete FCS/airframe configuration, the open-loop Bode characteristics of θ/θ_e (using the simplified pilot model) can be obtained from the following expression:

$$\theta/\theta_e = K_p e^{-0.3} s(\theta/F_S) \tag{1}$$

To apply the performance standards in the closed-loop analysis, it is necessary to have a method to convert the open-loop (θ/θ_e) Bode characteristics into the closed-loop (θ/θ_e) characteristics. One of the simplest and most illustrative methods for effecting this transformation is by plotting the (θ/θ_e) amplitude vs phase on a Nichols chart. A Nichols chart is simply a plot showing lines of constant closed-loop amplitude and phase on a grid of open-loop amplitude vs

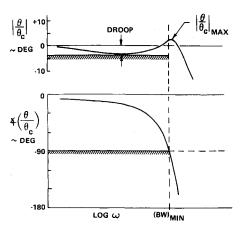


Fig. 5 Tracking performance standards used in the analysis.

phase. Figure 6 shows a Nichols chart, with the hatched boundaries representing the performance standards of Fig. 5.

If a piece of transparent paper is overlayed on a Nichols chart, $|\theta/\theta_e|$ can be plotted on the overlay vs $\not\prec$ (θ/θ_e) . The effects of changing K_p can then be seen by sliding the amplitude-phase overlay vertically on the Nichols chart. The value of K_p should be chosen so that the performance standards are just barely met. [The overlay should be positioned vertically so that the amplitude-phase curve is just barely above the hatched boundaries for all frequencies less than $(BW)_{\min}$.] If this is done, there are three basic types of amplitude-phase overlays which can result, as shown by curves A, B, C of Fig. 6.

The form of the pilot's compensation for each curve in Fig. 6 is discussed below:

Curve A (limited by the bandwidth and droop requirements simultaneously): No value of the pilot's compensation in the assumed form will reduce the closed-loop resonance (+3 dB) without either increasing the droop or decreasing the bandwidth. In this case, the pilot can be expected to use no compensation at all

$$F_S/\theta_e = K_p e^{-0.3} s \tag{2}$$

Curve B (limited by the bandwidth requirements alone): In this case, the closed-loop resonance (+12 dB) can be reduced by using lead compensation:

$$F_s/\theta_e = K_p e^{-0.3} s[(\tau_{p_1} s + 1)/(\tau_{p_2} s + 1)]$$
 $\tau_{p_1} > \tau_{p_2}$ (3)

Lead compensation will cause the lower part of the curve to shift upward and to the right and may cause it to flatten somewhat. (Curve B will become shaped more like curve A.) The compensated curve can now be shifted downward to reduce the resonance without reducing the bandwidth below $(BW)_{\min}$. The droop will increase, however. The amount of lead resulting in the least resonance will occur when the bandwidth can be made exactly equal to $(BW)_{\min}$ and the droop exactly equal to -3 dB simultaneously (similar to curve A).

Curve C (limited by the droop requirements alone): In this case, the closed-loop resonance (+12 dB) can be reduced by using lag compensation:

$$F_s/\theta_e = K_p e^{-0.3} s \left[(\tau_{p_1} s + 1/(\tau_{p_2} s + 1)) \right] \qquad \tau_{p_2} > \tau_{p_1}$$
 (4)

Lag compensation will cause the lower part of the curve to shift downward and to the left and also will cause it to steepen. (Curve C will become shaped more like curve A.) This will result in a reduction in the resonance. The amount of lag resulting in the least resonance will occur when the bandwidth can be made exactly equal to $(BW)_{\min}$ and the droop exactly equal to -3 dB simultaneously (similar to curve A).

Note that the pilot probably would not use any compensation at all if curves B and C had a resonance of less than 0 dB, using K_p alone.

With the form of the compensation determined for a given configuration, it remains to determine the "optimum" choice of τ_{p_1} and τ_{p_2} . To accomplish this, it is necessary first to define what is meant by the "optimum" compensation. In general terms, it is the compensation that will minimize the closed-loop resonance while still achieving the performance standards. As shown in the final report for the present investigation, an ear-optimum lead compensation is obtained when the pilot uses pure lead ($\tau_{p_2} = 0$). The report also shows that the near-optimum lag compensation is obtained when $1/\tau_{p_2}$ and $1/\tau_{p_1}$ are chosen so that $(BW)_{\min}$ is centered (logarithmically) between them, so that

$$(\tau_{p_1} \cdot \tau_{p_2})^{-1/2} = (BW)_{\min} \tag{5}$$

Amplitude-phase curves for the "optimum" lag compensation discussed previously are shown in Fig. 7 for various values of (τ_{p2}/τ_{p1}) . Also shown in this figure are amplitude-phase curves for "optimum" lead compensation $(\tau_{p2} = 0)$.

Using Fig. 7, the pilot compensation can be determined by choosing the value of τ_{p_1} or (τ_{p_2}/τ_{p_1}) which will cause the bandwidth to equal $(BW)_{\min}$ exactly and the maximum droop to equal -3 dB exactly. This will result in the smallest resonance, while still meeting the performance standards.

Example of a Configuration Having Low ω_{SP}

The example chosen is a 350-knot configuration from the present experiment having a low short-period frequency, good damping, and negligible control-system dynamics (configuration 6C). The open-loop Bode characteristics for this configuration plus the uncompensated pilot are shown in Fig. 8 (for an arbitrary value of K_p).

The effects of changing K_p can be seen by overlaying a plot of $|\theta/\theta_e|$ vs $\not<(\theta/\theta_e)$ on a Nichols chart. Such an overlay is shown in Fig. 9, positioned on the Nichols chart in an attempt to meet the performance standards. It is obvious from the figure that a bandwidth of 3.5 rad/sec can never be achieved using K_p alone, without driving the system unstable. In this case, the pilot must use lead compensation to increase the bandwidth.

Using Fig. 7, the "optimum" pilot compensation can be determined by choosing the value of τ_{p_1} which will cause the bandwidth to equal 3.5 rad/sec exactly and the maximum droop to equal -3 dB exactly. This will result in achieving the performance standards with the smallest resonance. The process can be accomplished very quickly by trial and error, if the amplitude-phase curve of Fig. 7 is graphically

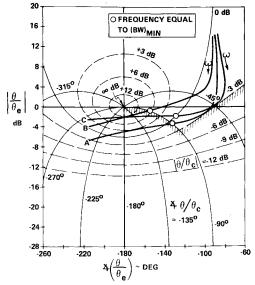


Fig. 6 Typical overlays of $|\theta/\theta_e|$ vs $\not<(\theta/\theta_e)$ on a Nichols

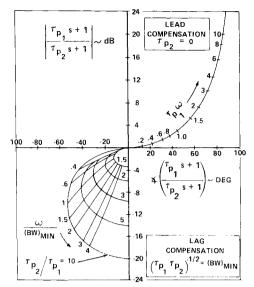


Fig. 7 Amplitude-phase plots for "optimum" pilot compensation.

added to the overlayed uncompensated curve of Fig. 9 for a few trial values of τ_{p_1} . The final compensated amplitude-phase curve is shown in Fig. 10, positioned on a Nichols chart so that the performance standards are met for $\omega \leq 3.5$ rad/sec. Comparison of Fig. 9 with Fig. 10 shows that the use of lead compensation has allowed the pilot to reduce the resonance to a negligible value, while maintaining a bandwidth of 3.5 rad/sec and a droop of -3 dB.

Factors Influencing Pilot Opinion

The parameters K_p , τ_{p_1} , τ_{p_2} , which the pilot would choose in "adapting" to a particular configuration, together with the resulting closed-loop performance characteristics, can be determined from the preceding analysis. It now remains to relate the primary pilot opinion factors to the various parameters determined from the analysis. On the basis of a detailed study of the pilot comments, the following relationships are offered:

- 1) PIO tendencies: It seems straightforward to relate the pilot's complaints of oscillatory tendencies to the closed-loop resonance $|\theta/\theta_c|_{\text{max}}$.
- 2) Pilot compensation: It would seem that the pilot's comments concerning his compensation are closely related to whether he has to generate phase lead or phase lag (over and above the phase lag caused by his 0.3-sec time delay). Since the phase characteristics are most important for frequencies in the vicinity of the bandwidth, it seems logical to describe

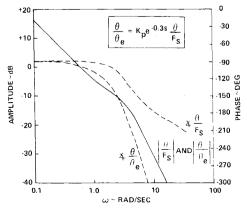


Fig. 8 Open-loop Bode characteristics for configuration 6C.

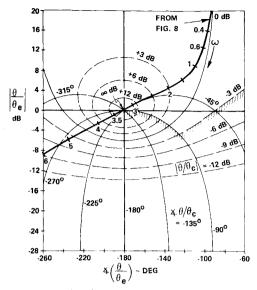


Fig. 9 Overlay of $|\theta/\theta_e|$ vs $\not < (\theta/\theta_e)$ on a Nichols chart (configuration 6C).

the pilot's compensation in terms of the following phase angle:

$$\swarrow_{pc} = \swarrow [(\tau_{p_1} s + 1)/(\tau_{p_2} s + 1)]_{\omega = (BW)_{\min}}$$
(6)

This phase angle can be determined from Fig. 7 for the particular value of (τ_{p2}/τ_{p1}) or τ_{p1} used. $\not\prec_{pe}$ will be positive for lead compensation and negative for lag compensation. Thus, when the pilot complains of having to "overdrive" the airplane, $\not\prec_{pe}$ will probably be positive. When he complains of having to "fly it smoothly" $\not\prec_{pe}$ will probably be pegative

of having to "fly it smoothly," $\not>_{pc}$ will probably be negative.

3) Stick forces: During the present experiment, the pilots often commented on the steady stick forces and the initial forces (or forces required for tracking). It is clear, from the comments, that the steady forces referred to are related to the steady-state stick force per g (F_s/n) . In selecting the elevator-to-stick-force gearing, the pilots insisted on a gearing that would permit pulling large load factors with reasonably steady forces. This often caused the initial forces (which probably are related to pilot gain) to be compromised. For this experiment, however, neither the values of F_s/n nor pilot gain appeared to have been far enough off-optimum to have any strong effect on over-all pilot opinion.

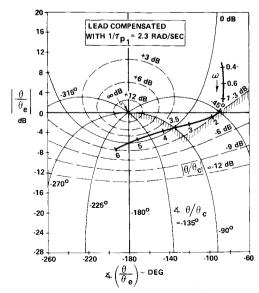


Fig. 10 Lead-compensated amplitude-phase curve overlayed on a Nichols chart (configuration 6C).

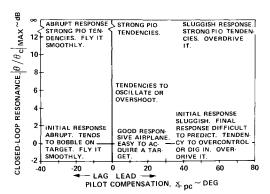


Fig. 11 Summary of pilot comment data as a function of closed-loop parameters.

IV. Application of the Analysis to the Results of the Present Experiment

Each configuration evaluated in the present experiment was analyzed using the techniques presented previously. In the following discussion, the pilot comments are explained in light of the analysis, and the pilot ratings are then correlated with the analysis parameters.

It should be mentioned that, for the 250-knot flight condition, the T-33 encounters buffet onset at n=2.5 to 3 g. There is evidence from the pilot comments that this factor caused the pilots to fly the 250-knot configurations somewhat less aggressively than the 350-knot configurations. This would suggest that the pilots decreased $(BW)_{\min}$ for the lowspeed cases. Analysis of the 350-knot data, using a $(BW)_{\min}$ of 3.5 rad/sec, resulted in values of $\not<_{pc}$ and $|\theta/\theta_c|_{\max}$ which correlated very well with the pilot comments. The pilot comments for the 250-knot configurations, however, were not as severe as the results of using $(BW)_{\min} = 3.5$ would indicate. A value of $(BW)_{\min}$ equal to 3 rad/sec was then used for the low-speed cases, which resulted in good correlation with the comments.

Correlation with Pilot Comments

A detailed study of the pilot comments³ showed that the trends in the pilot comments, for various combinations of short-period and control-system dynamics, could be nicely explained in terms of the parameters $\not\leftarrow_{pc}$ and $|\theta/\theta_c|_{max}$. Of course, there are aspects of the comments for individual configurations which are not completely accounted for, but it must be remembered that the purpose of the analysis is to explain the causes of the more important piloting difficulties,

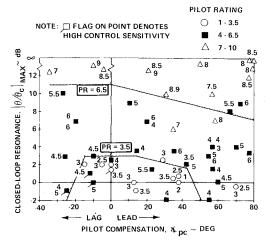


Fig. 12 Correlation of pilot M ratings with closed-loop parameters.

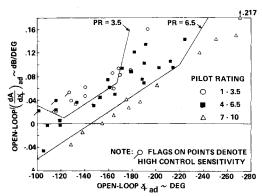


Fig. 13 Correlation of pilot M ratings with simplified analysis parameters.

and not to show exactly how the pilot flies the airplane. With these ideas in mind, Fig. 11 summarizes the pilot comments associated with various combinations of $\not\lt_{pc}$ and $|\theta/\theta_c|_{max}$.

For some configurations (usually those with negative values of χ_{pc}), the pilot comments indicate tendencies to oscillate or "bobble" on target which are more severe than the analysis predicts. These comments are usually accompanied by complaints of sensitivity to inadvertent control inputs. One possible explanation of the bobbling tendencies is that the high pitch-acceleration sensitivity of these configurations caused high-frequency inadvertent inputs. If this control sensitivity is interpreted in terms of the peak Bode amplitude $|\ddot{\theta}/F_S|_{\text{max}}$, it can be computed from θ/F_S Bode plots. For those configurations having $|\ddot{\theta}/F_S|_{\text{max}}$ greater than 0.5 (rad/sec²)/lb, the comments indicate that the bobbling tendencies and initial control sensitivity were enough of a problem to cause the pilot to downrate the configuration somewhat.

Correlation with Pilot Ratings

The pilot rating data from the present experiment are shown in Fig. 12 (for one pilot). The 3.5 and 6.5 pilot rating boundaries shown on the figure are based primarily on the ratings of both pilots, although additional factors were considered. In determining the boundaries, for example, more weight was given to those configurations with well substantiated ratings and consistent comments. Those configurations with additional problems, such as high control sensitivity (flagged symbols), or those rated by only one pilot and seemingly inconsistent relative to the other configurations within the same short-period group were given less weight.

The pilot rating data separate quite nicely into three regions consistent with the pilot rating boundaries shown and therefore correlate with the closed-loop parameters χ_{pc} and $|\theta/\theta_c|_{\rm max}$. There are some data points that violate the pilot rating boundaries, but, in most cases, the ratings of the other pilot tend to offset any discrepancy with the boundary. Considering the large variety of configurations represented on this figure and the potential sensitivity of the data points to the manner in which the pilot performs the required fighter tasks, the correlation of pilot ratings with the closed-loop parameters is considered good.

V. Proposed Design Criteria

Using the techniques described in this paper, the closed-loop parameters $\not\succsim_{\it pc}$ and $|\theta/\theta_{\it c}|_{\rm max}$ can be determined readily for any airplane whose maneuvering characteristics are available in the form of Bode plots. Comparison of these two parameters with the boundaries of Fig. 12 will then indicate the suitability of the airplane's maneuvering characteristics for the "combat" phase of the fighter mission. In apply-

ing the criterion, it is recommended that $(BW)_{\min} = 3.5$ rad/sec be used.

Simplified Criterion

The criterion discussed previously relates directly to the pilot's difficulties in performing the mission; it is applicable to airplanes having complex FCS dynamics and is not dependent on how the control system is mechanized. However, it would be desirable to have a "quicky" method for making initial design estimates. Referring to Figs. 9 and 10, it can be seen that the amount of pilot compensation which must be applied is related to the open-loop phase of the uncompensated pilot plus airplane, at $\omega = (BW)_{\min}$. This phase angle, which is the phase of the airplane plus pilot delay, is defined as follows:

$$\not \leq_{ad} = \not \leq [(\theta/F_S)e^{-0.3j\omega}]$$
 at $\omega = (BW)_{\min}$ (7)

As explained in more detail in the final report,³ the closed-loop resonance is related not only to the phase but also to the slope of the uncompensated amplitude-phase curve at $\omega = (BW)_{\min}$. The slope of the uncompensated curve is defined as follows:

$$\left(\frac{dA}{d\mathcal{X}}\right)_{ad} = \frac{d\left|(\theta/F_S)e^{-0.3j\omega}\right|}{d\left[\mathcal{X}(\theta/F_S)e^{-0.3j\omega}\right]} \quad \text{at } \omega = (BW)_{\min} \quad (8)$$

Thus, it would appear that the pilot compensation required and the closed-loop resonance are determined, in a crude sense, by the parameters $\not \subset_{ad}$ and $(dA/d \not\subset)_{ad}$.

These two open-loop parameters can be determined easily by making three measurements from the θ/F_s Bode plots at $\omega = (BW)_{\min}$: $d|\theta/F_s|/d(\log\omega)$ (in dB/decade), $d(\not \in \theta/F_s)/d(\log\omega)$ (in deg/decade), and $\not \in \theta/F_s$ (in degrees). The following relationships between these three measurements and $(dA/d\not \downarrow)_{ad}$ are derived in the final report³:

$$\begin{array}{lll}
\stackrel{\bigstar}{}_{ad} &= \left[\stackrel{\bigstar}{} (\theta/F_S)_{(BW)_{\min}} - 17.2 \; (BW)_{\min} \right] & \text{deg} & (9) \\
\left(\frac{dA}{d \stackrel{\bigstar}{}} \right)_{ad} &= \left[\frac{(d|\theta/F_S|/d \; (\log\omega))_{(BW)_{\min}}}{(d(\stackrel{\bigstar}{}\theta/F_S)/d (\log\omega))_{(BW)_{\min}} - 39.6 \; (BW)_{\min}} \right] \\
& \text{dB/deg} & (10)
\end{array}$$

Using Eqs. (9) and (10), the parameters $\not \in_{ad}$ and $(dA/d \not \in_{ad})$ were computed from Bode plots for each FCS/short-period configuration evaluated in the present program. The pilot ratings associated with each combination of $\not \in_{ad}$ and $(dA/d \not \in_{ad})$ are shown in Fig. 13 (for one pilot). As with the more general criterion, it was necessary to use $(BW)_{\min}$ of 3.5 rad/sec for the high-speed data and 3.0 rad/sec for the low-speed data. Boundaries were drawn on the plots, which separate the data very nicely into three bands of pilot ratings. These boundaries form a simple design criterion. In applying the criterion, it is recommended that $(BW)_{\min} = 3.5 \text{ rad/sec}$ be used.

Additional Considerations

Although the criteria discussed previously adequately describe the more important aspects of fighter longitudinal flying qualities for the "combat" flight phase, there are some other factors that must be considered separately.

For example, the elevator-to-stick-force gearing must be selected to provide good values of F_s/n without causing difficulties due to tracking forces or control sensitivity. The results of the present experiment indicate that the appropriate values of F_s/n for a fighter are adequately set by the high n/α requirements of MIL-F-8785B.^{8,9} The requirements show a range of 3.5 to 9.3 lb/g for satisfactory values of F_s/n (based on a limit load factor of 7 g).

Control sensitivity, $|\ddot{\theta}/F_S|_{\rm max}$, can cause difficulties if it becomes too large. Although not well documented, values of sensitivity greater than 0.5 (rad/sec²)/lb are likely to cause problems.

Another factor that is not handled by the criteria is the airplane's response to atmospheric turbulence. Turbulence is a complex subject in itself, and its effects were not treated in detail in the present experiment. The subject is mentioned to remind the designer that there are many ways to achieve the desired θ/F_S dynamics (prefilters, feedback loops, compensation networks, etc.), and a particular design that has good θ/F_S characteristics will not necessarily result in good response to turbulence.

VI. Conclusions

- 1) For the "combat" phase of a fighter's mission, those tasks that require precise control of pitch attitude are among the most critical from the standpoint of longitudinal flying qualities.
- 2) The results of this experiment show that the dynamic modes of the flight-control system can cause serious flying qualities problems, even if the short-period mode is well behaved.
- 3) The C^* criterion does not adequately account for control-system dynamics.
- 4) The pilot-in-the-loop analysis techniques presented in this report can be used effectively to describe the pilot's difficulties in precision tracking and to provide insight into the manner in which the pilot flies the airplane. These techniques are shown to be applicable to a wide range of control-system and short-period dynamics.
- 5) A criterion, based on the pilot-in-the-loop analysis, can be used for the design of good fighter maneuvering characteristics, provided that F_S/n and control sensitivity are kept within reasonable bounds. This criterion appears applicable to airplanes having highly augmented flight-control systems, as well as unaugmented airplanes.
- 6) A simplified version of the criterion, based on open-loop parameters, can be used to provide the flight-control system designer with a "quicky" method for estimating the effects of his control-system design on the airplane's flying qualities.

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